Calculus Warm Up #3-3

No calculator

Same region as described in your Monday WU...

The region in the first quadrant bounded by:

$y = 2\sqrt{x}$, $y = 6$, and the y-axis.

1) Write an integral that gives the volume of the solid when the region is rotated about the line $y = 7$.

2) Write an integral that gives the volume for the solid generated by rectangular cross sections taken perpendicular to the y-axis, where the height of the rectangle is three times the length of the base which is in the xy plane.

*Don't need to evaluate these integrals.

Quick Review

① $\sum_{n=0}^{\infty} \frac{1}{2^n}$ $\left(\frac{1}{2}\right)^n$ Geometric

$\sum_{n=0}^{\infty} \frac{n}{2^n}$ Not Geom.

② $\sum_{n=1}^{\infty} \frac{1}{n^3}$ p-Series

$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ Not p-Ser.

What about:

$\sum_{n=1}^{\infty} \frac{2}{n^3}$ or $\sum_{n=1}^{\infty} \frac{1}{2n^3}$?

$2 \sum \frac{1}{n^3}$ $\frac{1}{2} \sum \frac{1}{n^3}$

For the Integral Test...

$$a_n = \frac{n}{(n^2+3)^2}$$ easy to integrate

$$a_n = \frac{n^2}{(n^2+3)^2}$$ not so easy

More Tools ...

10.4 Comparison Tests

Direct Comparison Test

Limit Comparison Test

Direct Comparison Test

for $0 \le a_n \le b_n$

① If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ Converges

larger

② If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges

smaller

Determine if the series Converges or Diverges:

$$\sum_{n=1}^{\infty} \frac{1}{2+3^n}$$

Compare to: $\sum_{n=1}^{\infty} \frac{1}{3^n}$

term by term:

		n=1	n=2	n=3	n=4
$b_n =$	$\frac{1}{3^n}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$
(smaller) $a_n =$	$\frac{1}{2+3^n}$	$\frac{1}{5}$	$\frac{1}{11}$	$\frac{1}{29}$	$\frac{1}{83}$

All outcomes meet criteria

$$0 \le a_n \le b_n$$

$$0 \le \frac{1}{2+3^n} \le \frac{1}{3^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{2+3^n}$$

Evaluate the easier:

$b_n = \frac{1}{3^n}$ $\sum_{n=1}^{\infty} \frac{1}{3^n}$ Geometric $r = \frac{1}{3}$

$0 < |r| < 1$

so Series converges.

$\therefore \sum_{n=1}^{\infty} \frac{1}{2+3^n}$ converges by the direct comparison test.

Direct Comparison Test for $0 \le a_n \le b_n$

① If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ Converges

larger

② If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges

smaller

Step 1: Establish $0 \le a_n \le b_n$ by directly comparing terms

Step 2: Evaluate the easy one for Convergence and see if it supports a conclusion based on the theorem.

Limit Comparison Test

For when a term by term comparison is not possible.

For $a_n > 0$, $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$

(L = positive #)
(real)

When the conditions are met, either both series will converge or they will both diverge.

Limit Comparison Test

Example: $\sum_{n=1}^{\infty} \frac{1}{5n+2}$

compare to: $\sum_{n=1}^{\infty} \frac{1}{n}$

First establish meets conditions:

$a_n > 0$, $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$

$$\lim_{n\to\infty} \frac{\frac{1}{5n+2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{5n+2} = \frac{1}{5} \checkmark$$

* Conditions are met so they do the same thing.

Since the comparison series is a divergent p-series (harmonic series, p=1), then our example series also diverges.

Take some time in your team to compare process for MC problems you missed.

* Start with

MC - A

MC- A

9) $f'(x) = \frac{1}{x+4+e^{-3x}} \cdot (1 - 3e^{-3x})$

chain rule

$f'(0) = \frac{1-3}{1+4} = -\frac{2}{5}$

10) f decreasing and concave down

$f' < 0$ $f'' < 0$

all under x-axis → $f < 0$

MC- A

17) f is concave down when f'' is negative

$$f'(x) = 2xe^x + 2e^x$$

$$f''(x) = 2xe^x + 2e^x + 2e^x$$

$$0 = 2e^x(x+2)$$

f'': -3 ⊖ | -2 | 0 ⊕

$$\boxed{x < -2}$$

19) $\int (2x+3)\,dx$

$y = x^2 + 3x + c$ ← plug in $(1,2)$

$$2 = 1 + 3 + c$$

$$c = -2 \longrightarrow y = x^2 + 3x - 2$$

23) Fundamental Th: $\left[\sin\left((x^2)^3\right)\right] \cdot \frac{d}{dx}\left[x^2\right] = 2x \sin x^6$

Example of the Fundamental Theorem at work:

$$\frac{d}{dx}\int_0^{x^2} 3t\,dt \longrightarrow \text{should} = 3x^2 \cdot \frac{d}{dx}\left[x^2\right]$$

$$= 3x^2 \cdot 2x$$

$$= 6x^3$$

$$= \frac{d}{dx}\left[\frac{3t^2}{2}\right]_0^{x^2}$$

$$= \frac{d}{dx}\left[\frac{3x^4}{2} - \frac{3(0)^2}{2}\right]$$

$$= 4 \cdot \frac{3x^3}{2} = 6x^3$$

27) $(1,2)$ on $f \longrightarrow$ maps to $(2,1)$ on g.

Inverses have reciprocal slopes

so $f'(1)$ will be reciprocal to $g'(2)$

$f'(x) = 3x^2+1$

$f'(1) = 4 \longrightarrow \therefore g'(2) = \frac{1}{4}$

28) $g'(x)>0 \rightarrow g$ is increasing

$g''(x)>0 \rightarrow g$ is concave up and g' (slopes) are increasing

Average slope on $4 < x < 5$

$= \frac{18-12}{5-4} = 6$

$\therefore$ slopes after $x=5$ are >6

$g(6) > 18+6$

$g(6) > 24$

HW: p. 591, # 1 - 33 odd